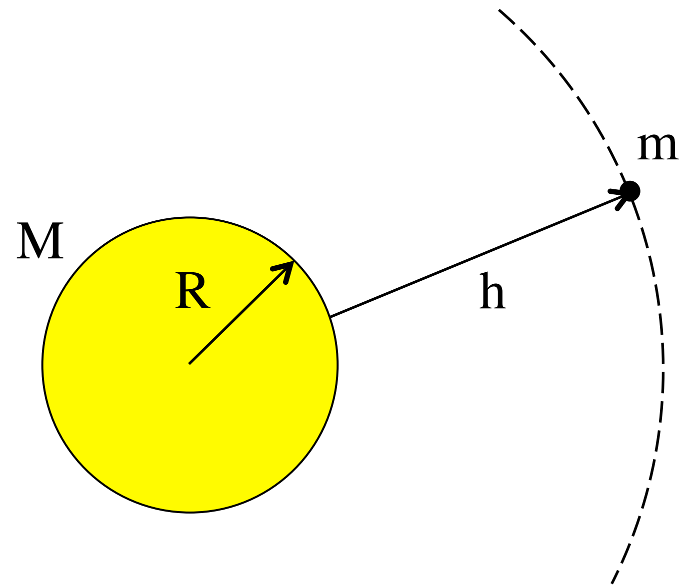
Problem 13.39

Consider the tiny satellite that is orbiting the earth.



a.) How long does it take to complete one orbit?

The trickiness here is to notice that the speed is equal to the distance traveled (the circumference in this case) divided by the period "T," or

$$T = \frac{2\pi(R+h)}{v}$$

All we need is the velocity. To get that, we can use N.S.L.:

$$\underline{\sum F_{centripetal}:}$$

$$\frac{GmM}{(R_e+h)^2} = ma_c = m\left(\frac{v^2}{R_e+h}\right)$$

$$\Rightarrow \qquad v = \left(\frac{GM}{R_e+h}\right)^{1/2}$$

So we can write:

$$T = \frac{2\pi(R+h)}{v}$$

$$= \frac{2\pi(R+h)}{\left(\frac{GM}{R_e+h}\right)^{1/2}} = 2\pi(R+h)\left(\frac{R_e+h}{GM}\right)^{1/2} = 2\pi(R+h)^{3/2}\left(\frac{1}{GM}\right)^{1/2}$$

$$= 2\pi\left(\left(6.37x10^{6}\ m\right)+\left(2.00x10^{5}\ m\right)\right)^{3/2}\left(\frac{1}{\left(6.67x10^{-11}\ N\bullet m^{2}/kg^{2}\right)\left(5.98x10^{24}\ kg\right)}\right)^{1/2}$$

$$= 5.30x10^{3}\ s \qquad (=1.47\ hr)$$

b.) Determine the satellite's speed.

$$v = \frac{2\pi(R+h)}{T}$$

$$= \frac{2\pi\left(\left(6.37x10^{6}\ m\right)+\left(2.00x10^{5}\ m\right)\right)}{\left(5.30x10^{3}\ s\right)}$$

$$= 7.79x10^{3}\ m/s$$

c.) What is the minimum energy required to put the satellite into orbit?

The only thing that is tricky is that although the satellite's initial take-off velocity will be zero (part of the energy required will be needed to boost it to take-off speed), the satellite will still have speed associated with the fact that it is sitting on a rotating earth. That speed has to be taken into consideration and is:

$$\begin{aligned} v_{earth} &= \frac{2\pi R_e}{T_{earth}} \\ &= \frac{2\pi\left(6.37x10^6 \text{ m}\right)}{(24 \text{ hr})\left(\frac{3600 \text{ s}}{\text{hr}}\right)} \\ &= 4.63x10^3 \text{ m/s} \end{aligned}$$

Putting everything together, and suppressing the units to save space, we can write:

$$\sum KE_1 + \sum U_1 + \sum W_{energyInput} = \sum KE_2 + \sum U_2$$

$$\frac{1}{2}m(v_e)^2 + \left(-\frac{GmM_e}{R_{earth}}\right) + W_{energyInput} = \frac{1}{2}m(v_{orbit})^2 + \left(-\frac{GmM_e}{(R_{earth}+h)}\right)$$

$$\Rightarrow \quad W_{energyInput} = -\frac{1}{2}m(v_e)^2 - \left(-\frac{GmM_e}{R_e}\right) + \frac{1}{2}m(v_{orbit})^2 + \left(-\frac{GmM_e}{(R_e+h)}\right)$$

$$= \frac{1}{2}m\left((v_{orbit})^2 - (v_e)^2\right) + GmM_e\left(\frac{1}{R_e} - \frac{1}{(R_e+h)}\right)$$

$$= \frac{1}{2}(200.)\left((7.79x10^3)^2 - (4.63x10^2)^2\right) +$$

$$+(6.67x10^{-11})(200)(5.98x10^{24})\left(\frac{1}{(6.37x10^6)} - \frac{1}{(6.57x10^6)}\right)$$

$$= 6.43x10^9 \text{ J}$$